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CALCULATION OF DEFORMATION IN A GLASS PLATE IN BENDING BY GRAVITY

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Based on the general IIKhS model, a method of calculation of strains and their ratios in different coordinates for a cantilever glass plate is proposed, which allows for optimization of the molding process and determination of the design parameters of the molded surfaces.

Among the most significant parameters in the development and optimization of the molding process are the time parameters, especially those known as after-effect, or, in other words, unbending of a sample under the effect of gravity and internal stresses. In this context, the variation of strains by different coordinates and their ratio, i.e., determination of the type of expression

$$\frac{\omega_x(\tau)}{\omega_y(\tau)},$$

where τ is the process duration, are of great importance.

In order to achieve this purpose, we used the plate clamping scheme shown in Fig. 1. In this case, the deflection is calculated for a cantilever-fixed plate loaded by a distributed gravity load. Then force q can be determined from the formula

$$q = h\rho g, \tag{1}$$

where h is the plate thickness; ρ is the glass density; g is the free fall acceleration.

Using the given scheme and the IKhS model [1]

$$\varepsilon(\tau) = \varepsilon(0) + \varepsilon_\eta(\tau) + \varepsilon_d(\tau),$$

where $\varepsilon(\tau)$ is the elongation of the middle layer; $\varepsilon(0)$ is the elastic component of deformation; $\varepsilon_\eta(\tau)$ is the viscous component; $\varepsilon_d(\tau)$ is the creep component, we try to obtain the dependence of the $\omega(\varepsilon(\tau))$ form for the respective coordinates.

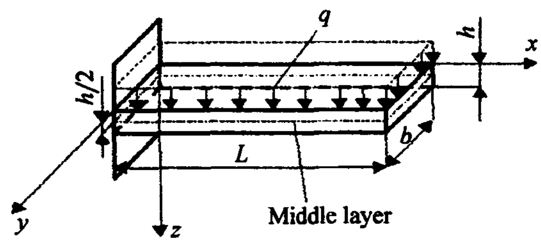

Fig. 1. Scheme of clamping and loading of the plate. L) Plate length, b) width; h) thickness; q) distributed gravity load.

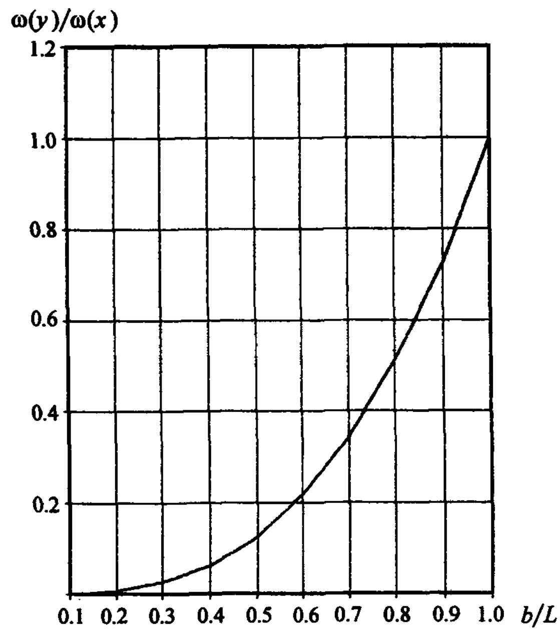

Fig. 2. Diagram of the variations in the deflection ratio $\omega(y)/\omega(x)$.

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The basic system of equations describing the elongation of the middle layer in deformation of the plate is as follows [2]:

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E};$$
$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E},$$

$$\begin{cases} \sigma_x = \dfrac{12z}{h^3}\mathrm{M}_x; \\ \sigma_y = \dfrac{12z}{h^3}\mathrm{M}_y, \end{cases} \quad (2)$$

where μ is the Poisson coefficient; σ_x and σ_y are stresses for the respective coordinates; E is the modulus of elongation.

The stresses which are part of the above expression can be determined from the following dependences:

$$\mathrm{M}_x = \frac{qx^2b}{2};\ \mathrm{M}_y = \frac{qy^2L}{2},$$

where M_x and M_y are the bending moments for the respective coordinates.

In the case of using the clamping scheme shown in Fig. 1, the moments can be calculated by the following formulas:

$$\begin{cases} \varepsilon_x = z\dfrac{\partial^2\omega}{\partial x^2}; \\ \varepsilon_y = z\dfrac{\partial^2\omega}{\partial y^2}. \end{cases} \quad (3)$$

In order to determine the form of dependence $\omega(\varepsilon)$, we use substitution of z from system (2) in system (3):

$$\begin{cases} \varepsilon_x = \dfrac{\sigma_x h^3}{12\mathrm{M}_x}\dfrac{\partial^2\omega}{\partial x^2}; \\ \varepsilon_y = \dfrac{\sigma_y h^3}{12\mathrm{M}_y}\dfrac{\partial^2\omega}{\partial y^2}. \end{cases} \quad (4)$$

The initial stresses σ are calculated from the equations:

$$\sigma_x = \frac{\mathrm{M}_x}{W_x};\quad \sigma_y = \frac{\mathrm{M}_y}{W_y};$$
$$W_x = \frac{J_x}{h/2};\quad W_y = \frac{J_y}{h/2},$$

where W_x and W_y are the moments of resistance of the beam section for the respective coordinates; J_x and J_y are the moments of inertia for the respective coordinates.

Now that all parameters involved in system (4) are determined, the unknown dependence can be obtained by direct integration of these equations after expressing the second derivative in explicit form. In order to determine the values of the integration constants C, it is necessary to use the boundary conditions typical of the given type of plate clamping, i.e., at $x = 0$, both the first derivative, and $\omega(\varepsilon)$ are equal to zero. As a result, the following equations were obtained which describe the process of plate deformation, taking into account the IKhS model:

$$\begin{cases} \omega(x) = \dfrac{qb}{2\sigma_x h^3}x^4\varepsilon_x; \\ \omega(y) = \dfrac{qL}{2\sigma_y h^3}y^4\varepsilon_y. \end{cases} \quad (5)$$

Since we need to determine the ratio of the deflections for different sizes of the plate, we will use the dependence derived from system (5):

$$\frac{\omega(\varepsilon_y)}{\omega(\varepsilon_x)} = \frac{\sigma_x}{\sigma_y}A^3\frac{\varepsilon_y}{\varepsilon_x},$$

where A is the ratio of the plate width to its length.

Based on this equation, calculations were performed for the following initial parameters: plate length of 0.1 m; width of 0.01 m, thickness of 0.004 m; temperature of the sample of 620°C; density of 2500 kg/m^3, deformation time of 1 sec. The calculation results are shown in Fig. 2

Thus, the ratio of the plate deflections for different coordinates depends only on the dimensions of the plate. The secondary curvature, i.e., the curvature along axis y has a significant effect only when the ratio between the width and the length of the plate exceeds 0.3, i.e., when the ratio of the deformation along axis y to the deformation along axis x is over 5%.

The proposed method can be used to optimize the processes of shaping (molding and pressing) of sheet glass, and to design shaping surfaces (dies and molds) for the known time parameters of the technological process, which will help to avoid flaws caused by unbending of articles under the effect of gravity.

REFERENCES

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